Published by Institute of Physics Publishing for SISSA/ISAS
Received: *August 1, 2002*
Accepted: *October 8, 2002*

Renormalization of the Planck mass for type-II superstrings on symmetric orbifolds

Emmanuel Kohlprath

Theoretical Physics Division, CERN
CH - 1211 Geneva 23, Switzerland
E-mail: Emmanuel.Kohlprath@cern.ch

Abstract: We compute the one-loop renormalization of the Planck mass for type II string theories compactified to four dimensions on symmetric orbifolds that preserve $\mathcal{N} = 2$ supersymmetry. Depending on the orbifold, the effect can be as large as to compete with the standard tree-level value.

Keywords: Renormalization Regularization and Renormalons, Superstrings and Heterotic Strings, D-branes.

JHEP10(2002)026

Contents

1. Introduction **1**

2. Background field method **2**

3. Conclusion **7**

1. Introduction

The weakness of the gravitational force can be rephrased as an enormous hierarchy between the Planck mass $M_{\rm P}$ and the scales characterizing non-gravitational interactions. In the context of supersymmetric Grand Unified Theories the hierarchy between the Planck scale $M_{\rm P}$ and the GUT scale $M_{\rm GUT}$ appears to be less dramatic (a mere 10^3), but is nevertheless puzzling.

The natural framework for trying to understand this ratio is obviously superstring theory, since it supposedly unifies all interactions. Such a theory contains a scale of its own, the string mass $M_{\rm s}$, which plays the role of a fundamental UV cut-off related to the finite size of quantum strings. It is quite natural to identify $M_{\rm s}$ with $M_{\rm GUT}$ (though not an absolute theoretical necessity) and, in this case, explaining the origin of the ratio $M_{\rm s}/M_{\rm P}$ becomes the problem.

Unfortunately, for closed strings and at tree level (or in string-loop perturbation theory), this ratio gets related to $\sqrt{\alpha_{\rm GUT}}$ in such a way that, when all factors are included, one finds $M_{\rm s} \approx 2 \times 10^{17}$ GeV, i.e. an order of magnitude too large a value for identification with $M_{\rm GUT} \approx 2 \times 10^{16}$ GeV. If we wish to insist on $M_{\rm s} \approx M_{\rm GUT}$, this leaves e.g. the possibilities of either considering open string theory, or going beyond the validity of the loop expansion (for other possibilities see [1]).

A suggestion in the latter direction was recently made in [2], where it was proposed that, in the infinite bare coupling (also known as compositeness) limit, loop corrections may explain $M_{\rm s}/M_{\rm P} \ll 1$ as a result of the different large-N behaviour of gravity and gauge loops in a theory with a large number N of particle species. The arguments given in [2] were quite heuristic since they assumed some UV completion of a toy model, making it UV-finite, as well as the existence of an arbitrarily large parameter N. In this paper we shall explore this suggestion (at one loop) in the context of superstring theories with $\mathcal{N} = 2$ space-time supersymmetry. A larger value of $\mathcal{N}$ would presumably induce no one-loop renormalization of $M_{\rm P}$, while the non-supersymmetric case, $\mathcal{N} = 0$, would be generally plagued by a large one-loop cosmological constant. Unfortunately, the models for which we shall be able to perform the calculation do not contain an arbitrary parameter like N. At the end of the paper we shall introduce models that do have this feature, but for which, at present, we are unable to carry out a full string-theory calculation.

JHEP10(2002)026

2. Background field method

The Einstein-Hilbert lagrangian reads (we use units in which $\hbar = c = 1$)

$$\mathcal{L} = \frac{1}{16\pi G}\sqrt{g}R = \frac{M_{\rm P}^2}{16\pi}\sqrt{g}R\,. \tag{2.1}$$

In [3] Kiritsis and Kounnas compute the one-loop renormalization of the coupling of the Einstein-Hilbert action (i.e. of Newton's constant G or of the Planck mass $M_{\rm P}$) for the compactification of type-II string theory (both IIA and IIB) on the symmetric $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold to 4 dimensions. First we notice that this compactification preserves $\mathcal{N} = 2$ space-time supersymmetry in 4 dimensions. If there were more unbroken supersymmetries, then there should be no one-loop correction to the Planck mass due to extra zero modes. We generalize their results in [3] using the background field method (see also [4] to [10]) to a large class of orbifold compactifications with $\mathcal{N} = 2$ space-time supersymmetry in 4 dimensions. We can choose the axes such that the point group generator for $\mathbb{Z}_N$ orbifolds has the form (see [11] and [12] for the theory of orbifolds)

$$\theta = \exp[2\pi i(v_2 J_{45} + v_3 J_{67} + v_4 J_{89})]\,. \tag{2.2}$$

The criterion for space-time supersymmetry is then

$$\pm v_2 \pm v_3 \pm v_4 = 0\,, \tag{2.3}$$

for some choice of the signs. If the v_i are otherwise non-vanishing then we have $\mathcal{N} = 2$ space-time supersymmetry. If one of the v_i's is zero, then we have $\mathcal{N} = 4$ space-time supersymmetry. For the $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifolds we have a second set of point-group generators

$$\varphi = \exp[2\pi i(w_2 J_{45} + w_3 J_{67} + w_4 J_{89})]\,. \tag{2.4}$$

The criterion for space-time supersymmetry then is

$$\pm v_2 \pm v_3 \pm v_4 = 0\,, \qquad \pm w_2 \pm w_3 \pm w_4 = 0\,, \tag{2.5}$$

for some choice of the signs. Only if $v_i = w_i = 0$ for the same $i = 2,3,4$ do we have $\mathcal{N} \geq 4$ space-time supersymmetry. The requirement that the point group acts crystallographically and the conditions (2.3) and (2.5) lead to the orbifold models listed in tables 1 and 2 (see e.g. [12]). Let us define the complex combinations

$$Z^i = \frac{1}{\sqrt{2}}(X^{2i} + iX^{2i+1})\,, \qquad i = 2,3,4 \tag{2.6}$$

for the compactified bosonic coordinates and

$$\Psi^i = \frac{1}{\sqrt{2}}(\psi^{2i} + i\psi^{2i+1})\,, \qquad i = 2,3,4 \tag{2.7}$$

Point group	(v_2, v_3, v_4)	s
$\mathbb{Z}_3$	$(1,1,-2)/3$	8
$\mathbb{Z}_4$	$(1,1,-2)/4$	12
$\mathbb{Z}_6 - I$	$(1,1,-2)/6$	32
$\mathbb{Z}_6 - II$	$(1,2,-3)/6$	24
$\mathbb{Z}_7$	$(1,2,-3)/7$	48
$\mathbb{Z}_8 - I$	$(1,2,-3)/8$	60
$\mathbb{Z}_8 - II$	$(1,3,-4)/8$	48
$\mathbb{Z}_{12} - I$	$(1,4,-5)/12$	128
$\mathbb{Z}_{12} - II$	$(1,5,-6)/12$	108

Table 1: Point-group generators for $\mathbb{Z}_N$ orbifolds.

Point group	(v_2, v_3, v_4)	(w_2, w_3, w_4)	s
$\mathbb{Z}_2 \times \mathbb{Z}_2$	$(1, 0, -1)/2$	$(0, 1, -1)/2$	6
$\mathbb{Z}_3 \times \mathbb{Z}_3$	$(1, 0, -1)/3$	$(0, 1, -1)/3$	56
$\mathbb{Z}_2 \times \mathbb{Z}_4$	$(1, 0, -1)/2$	$(0, 1, -1)/4$	42
$\mathbb{Z}_4 \times \mathbb{Z}_4$	$(1, 0, -1)/4$	$(0, 1, -1)/4$	210
$\mathbb{Z}_2 \times \mathbb{Z}_6 - I$	$(1, 0, -1)/2$	$(0, 1, -1)/6$	102
$\mathbb{Z}_2 \times \mathbb{Z}_6 - II$	$(1, 0, -1)/2$	$(1, 1, -2)/6$	134
$\mathbb{Z}_3 \times \mathbb{Z}_6$	$(1, 0, -1)/3$	$(0, 1, -1)/6$	272
$\mathbb{Z}_6 \times \mathbb{Z}_6$	$(1, 0, -1)/6$	$(0, 1, -1)/6$	1190

Table 2: Point-group generators for $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifolds.

for the fermions. The orbifold twist acts as

$$Z^i(\sigma + 2\pi) = e^{2\pi i v_i} Z^i(\sigma) \tag{2.8}$$

on the bosons and as

$$\tilde{\Psi}^i(\sigma + 2\pi) = e^{2\pi i(v_i+\nu)} \tilde{\Psi}^i(\sigma) \tag{2.9}$$

on the fermions, where $\nu = 0$ in the Ramond sector and $\nu = 1/2$ in the Neveu-Schwarz sector. For the models of tables 1 and 2, we therefore find the following partition function for a flat background generalizing the $\mathbb{Z}_2 \times \mathbb{Z}_2$ result of [3] (for $\mathbb{Z}_N$ orbifolds set $M = 1$):

$$\begin{aligned} Z(\tau,\bar\tau) = {} & \frac{1}{4NM}\frac{1}{\mathrm{Im}\,\tau|\eta|^4} \sum_{\alpha,\beta,\bar\alpha,\bar\beta=0}^{1} \sum_{h_1,g_1=0}^{N-1} \sum_{h_2,g_2=0}^{M-1} (-)^{\alpha+\beta+\alpha\beta}(-)^{\bar\alpha+\bar\beta+\bar\alpha\bar\beta} \times \\ & \times Z_2\begin{bmatrix} h_1v_2{+}h_2w_2 \\ g_1v_2{+}g_2w_2 \end{bmatrix} Z_3\begin{bmatrix} h_1v_3{+}h_2w_3 \\ g_1v_3{+}g_2w_3 \end{bmatrix} Z_4\begin{bmatrix} h_1v_4{+}h_2w_4 \\ g_1v_4{+}g_2w_4 \end{bmatrix} \times \\ & \times \frac{\theta\begin{bmatrix} \alpha/2 \\ \beta/2 \end{bmatrix}}{\eta} \frac{\theta\begin{bmatrix} \alpha/2{+}h_1v_2{+}h_2w_2 \\ \beta/2{+}g_1v_2{+}g_2w_2 \end{bmatrix}}{\eta} \frac{\theta\begin{bmatrix} \alpha/2{+}h_1v_3{+}h_2w_3 \\ \beta/2{+}g_1v_3{+}g_2w_3 \end{bmatrix}}{\eta} \frac{\theta\begin{bmatrix} \alpha/2{+}h_1v_4{+}h_2w_4 \\ \beta/2{+}g_1v_4{+}g_2w_4 \end{bmatrix}}{\eta} \times \\ & \times \frac{\bar\theta\begin{bmatrix} \bar\alpha/2 \\ \bar\beta/2 \end{bmatrix}}{\bar\eta} \frac{\bar\theta\begin{bmatrix} \bar\alpha/2{+}h_1v_2{+}h_2w_2 \\ \bar\beta/2{+}g_1v_2{+}g_2w_2 \end{bmatrix}}{\bar\eta} \frac{\bar\theta\begin{bmatrix} \bar\alpha/2{+}h_1v_3{+}h_2w_3 \\ \bar\beta/2{+}g_1v_3{+}g_2w_3 \end{bmatrix}}{\bar\eta} \frac{\bar\theta\begin{bmatrix} \bar\alpha/2{+}h_1v_4{+}h_2w_4 \\ \bar\beta/2{+}g_1v_4{+}g_2w_4 \end{bmatrix}}{\bar\eta}, \end{aligned} \tag{2.10}$$

where $Z_i[{}^{h/2}_{g/2}]$ is the partition function of the complex bosons Z^i with twists (h, g). We have

$$Z_i\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \frac{\Gamma(2,2)}{|\eta|^4}\,, \tag{2.11}$$

where $\Gamma(2, 2)$ is the (2,2) lattice sum, and

$$Z_i\begin{bmatrix} h/2 \\ g/2 \end{bmatrix} = \left| \eta(\tau) \left[\theta\begin{bmatrix} \frac{1-h}{2} \\ \frac{1-g}{2} \end{bmatrix} (0,\tau) \right]^{-1} \right|^2 \tag{2.12}$$

JHEP10(2002)026

for $(h,g) \neq (0,0)$ (see e.g. [11]). We introduce a constant background curvature parametrized by $\mathcal{R}$ by the perturbation

$$\int d^2 z\, \mathcal{R}[I^3 + :\psi^1 \psi^2:][\bar{I}^3 + :\tilde{\psi}^1 \tilde{\psi}^2:] \tag{2.13}$$

of the sigma model action, where $I^i = k\,\mathrm{Tr}[\sigma^i g^{-1} \partial g]$ and $g = \exp[i\sigma \cdot x/2]$. Let Q be the momentum lattice associated to the left-moving U(1) current $:\psi^1\psi^2:$ I the charge lattice of the left-moving U(1) current associated to I_3 and let $\bar{Q}$ and $\bar{I}$ be defined in terms of the right-moving currents. For a pair of dimensions the 2 dimensional Majorana-Weyl world-sheet spinors are given by one complex fermion

$$\psi = \frac{1}{\sqrt{2}}(\psi_1 + i\psi_2)\,, \qquad \bar{\psi} = \frac{1}{\sqrt{2}}(\psi_1 - i\psi_2)\,. \tag{2.14}$$

The fermions can be bozonized (see [11]) by $\psi = e^{iH}$ and $\bar{\psi} = e^{-iH}$ in such a way that $i\partial H = :\psi\bar{\psi}:$. The operator Q acts as the fermion-number operator. The undeformed partition function can be written as

$$Z = \mathrm{Tr}\left(q^{L_0} \bar{q}^{\bar{L}_0}\right)\,, \qquad q = e^{2\pi i \tau} \tag{2.15}$$

where

$$L_0 = \frac{1}{2} Q^2 + \frac{I^2}{k} + \cdots\,, \qquad \bar{L}_0 = \frac{1}{2} \bar{Q}^2 + \frac{\bar{I}^2}{k} + \cdots\,, \tag{2.16}$$

where the dots stand for operators that do not involve $I, \bar{I}, Q, \bar{Q}$. In the heterotic string theory the constant background field $\mathcal{R}$ transforms L_0 and $\bar{L}_0$ to

$$L_0' - L_0 = \bar{L}_0' - \bar{L}_0 = (Q+I) + \frac{\sqrt{1 + k(k+2)\mathcal{R}^2} - 1}{2} \left[\frac{(Q+I)^2}{k+2} + \frac{I^2}{k}\right]^2 . \tag{2.17}$$

Let us expand the partition function in a power series in $\mathcal{R}$

$$Z(\mathcal{R}) = \sum_{n=0}^{\infty} \mathcal{R}^n Z_n\,. \tag{2.18}$$

In string theory the partition function is already the generating functional of the connected Green functions (this is in contrast to field theory where one would have to take the logarithm) and the one-loop correction to the Einstein-Hilbert action in the effective action is therefore given by

$$Z_1^{\mathrm{het}} = -4\pi\, \mathrm{Im}\,\tau \langle (Q+I)\bar{I}\rangle\,. \tag{2.19}$$

As $\langle \bar{I} \rangle = 0$ on any genus Riemann surface, we find that the Planck mass is not renormalized in perturbation theory for heterotic backgrounds with $\mathcal{N} \geq 1$ space-time supersymmetry (see also [3] and [13] to [15]). In type-II string theory the renormalization of the Planck mass is given by

$$Z_1^{\mathrm{II}} = -2\pi\, \mathrm{Im}\,\tau \langle (Q+I)(\bar{Q}+\bar{I})\rangle\,. \tag{2.20}$$

JHEP10(2002)026

The contribution of the pair of (transverse) world-sheet fermions that correspond to the 4-dimensional space to the partition function is given by (see [11])

$$Z^{\alpha}_{\beta}(\tau) = \mathrm{Tr}_{\alpha}\left[q^H e^{i\pi Q\beta}\right] = \frac{\theta\left[\begin{smallmatrix}\alpha/2\\ \beta/2\end{smallmatrix}\right](0,\tau)}{\eta(\tau)}\,. \tag{2.21}$$

This was also used to derive (2.10). A similar computation leads to

$$\mathrm{Tr}_{\alpha}\left[q^H e^{i\pi Q\beta} Q\right] = \frac{1}{\eta(\tau)}\frac{1}{2\pi i}\frac{\partial}{\partial\nu}\theta\begin{bmatrix}\alpha/2\\ \beta/2\end{bmatrix}(\nu,\tau)\Bigg|_{\nu=0}\,. \tag{2.22}$$

To compute (2.20) we therefore can simply replace the corresponding left and right theta functions in (2.10) by their derivative. We can use the following Riemann identity

$$\begin{aligned}\frac{1}{2}\sum_{a,b=0}^{1}(-)^{\alpha+\beta+\alpha\beta}\theta\begin{bmatrix}\alpha/2\\ \beta/2\end{bmatrix}(\nu,\tau)\theta\begin{bmatrix}\frac{\alpha+h_1}{2}\\ \frac{\beta+g_1}{2}\end{bmatrix}(0,\tau)\theta\begin{bmatrix}\frac{\alpha+h_2}{2}\\ \frac{\beta+g_2}{2}\end{bmatrix}(0,\tau)\theta\begin{bmatrix}\frac{\alpha-h_1-h_2}{2}\\ \frac{\beta-g_1-g_2}{2}\end{bmatrix}(0,\tau) =\\ = \theta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}\left(\frac{\nu}{2},\tau\right)\theta\begin{bmatrix}\frac{1-h_1}{2}\\ \frac{1-g_1}{2}\end{bmatrix}\left(\frac{\nu}{2},\tau\right)\theta\begin{bmatrix}\frac{1-h_2}{2}\\ \frac{1-g_2}{2}\end{bmatrix}\left(\frac{\nu}{2},\tau\right)\theta\begin{bmatrix}\frac{1+h_1+h_2}{2}\\ \frac{1+g_1+g_2}{2}\end{bmatrix}\left(\frac{\nu}{2},\tau\right).\end{aligned} \tag{2.23}$$

For $\nu = 0$ this shows that the partition function (2.10) vanishes and that we have at least one unbroken supersymmetry. We find

$$\begin{aligned}-2\pi\,\mathrm{Im}\,\tau\langle Q\bar{Q}\rangle = {} & \frac{1}{NM}\frac{-2\pi\,\mathrm{Im}\,\tau}{\mathrm{Im}\,\tau|\eta|^4}\sum_{h_1,g_1=0}^{N-1}\sum_{h_2,g_2=0}^{M-1}\frac{1}{2\pi i\eta^4}\frac{1}{2\pi i\bar{\eta}^4}\times\\ & \times Z_2\begin{bmatrix}h_1v_2+h_2w_2\\ g_1v_2+g_2w_2\end{bmatrix} Z_3\begin{bmatrix}h_1v_3+h_2w_3\\ g_1v_3+g_2w_3\end{bmatrix} Z_4\begin{bmatrix}h_1v_4+h_2w_4\\ g_1v_4+g_2w_4\end{bmatrix}\times\\ & \times\frac{\partial}{\partial\nu}\Bigg[\theta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}\left(\frac{\nu}{2},\tau\right)\theta\begin{bmatrix}\frac{1}{2}-h_1v_2-h_2w_2\\ \frac{1}{2}-g_1v_2-g_2w_2\end{bmatrix}\left(\frac{\nu}{2},\tau\right)\times\\ & \qquad\times\theta\begin{bmatrix}\frac{1}{2}-h_1v_3-h_2w_3\\ \frac{1}{2}-g_1v_3-g_2w_3\end{bmatrix}\left(\frac{\nu}{2},\tau\right)\theta\begin{bmatrix}\frac{1}{2}-h_1v_4-h_2w_4\\ \frac{1}{2}-g_1v_4-g_2w_4\end{bmatrix}\left(\frac{\nu}{2},\tau\right)\Bigg]\Bigg|_{\nu=0}\times\\ & \times\frac{\partial}{\partial\bar{\nu}}\Bigg[\bar{\theta}\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}\left(\frac{\bar{\nu}}{2},\tau\right)\bar{\theta}\begin{bmatrix}\frac{1}{2}-h_1v_2-h_2w_2\\ \frac{1}{2}-g_1v_2-g_2w_2\end{bmatrix}\left(\frac{\bar{\nu}}{2},\tau\right)\times\\ & \qquad\times\bar{\theta}\begin{bmatrix}\frac{1}{2}-h_1v_3-h_2w_3\\ \frac{1}{2}-g_1v_3-g_2w_3\end{bmatrix}\left(\frac{\bar{\nu}}{2},\tau\right)\bar{\theta}\begin{bmatrix}\frac{1}{2}-h_1v_4-h_2w_4\\ \frac{1}{2}-g_1v_4-g_2w_4\end{bmatrix}\left(\frac{\bar{\nu}}{2},\tau\right)\Bigg]\Bigg|_{\bar{\nu}=0}\,.\end{aligned} \tag{2.24}$$

Because of

$$\theta\begin{bmatrix}1/2\\ 1/2\end{bmatrix}(0,\tau) = 0\,, \tag{2.25}$$

the partial derivatives with respect to ν and $\bar{\nu}$ have to act on the first theta functions. From the N^2M^2 possible sectors, those do not contribute where either

$$h_1v_2 + h_2w_2 = g_1v_2 + g_2w_2 = 0 \qquad \mathrm{mod}1 \tag{2.26}$$

$$\text{or}\qquad h_1v_3 + h_2w_3 = g_1v_3 + g_2w_3 = 0 \qquad \mathrm{mod}1 \tag{2.27}$$

$$\text{or}\qquad h_1v_4 + h_2w_4 = g_1v_4 + g_2w_4 = 0 \qquad \mathrm{mod}1\,. \tag{2.28}$$

JHEP10(2002)026

All sectors that contribute give the same contribution. This is because they are all twisted, and in this case the contribution of the twisted bosons cancels the one of the twisted fermions (see (2.12)). Let s be the number of contributing sectors. The different values of s are summarized in tables 1 and 2. We are left with

$$-2\pi\,\mathrm{Im}\,\tau\langle Q\bar{Q}\rangle = \frac{s}{NM}\frac{-2\pi\,\mathrm{Im}\,\tau}{\mathrm{Im}\,\tau|\eta|^4}\frac{1}{2\pi i\eta}\frac{\partial}{\partial\nu}\,\theta\begin{bmatrix}1/2\\1/2\end{bmatrix}(\nu/2,\tau)\bigg|_{\nu=0}\times \\ \times\frac{1}{2\pi i\bar{\eta}}\frac{\partial}{\partial\bar{\nu}}\,\bar{\theta}\begin{bmatrix}1/2\\1/2\end{bmatrix}(\bar{\nu}/2,\tau)\bigg|_{\bar{\nu}=0}. \tag{2.29}$$

Using

$$\partial_\nu\theta\begin{bmatrix}1/2\\1/2\end{bmatrix}(\nu,\tau)\bigg|_{\nu=0} = -2\pi\eta(\tau)^3, \qquad \frac{\partial}{\partial\nu} = \frac{1}{2}\frac{\partial}{\partial\frac{\nu}{2}}, \tag{2.30}$$

we arrive at

$$-2\pi\,\mathrm{Im}\,\tau\langle Q\bar{Q}\rangle = s\frac{\pi}{2NM}. \tag{2.31}$$

Multiplying with the fundamental domain integral

$$\int_{\mathcal{F}}\frac{d^2\tau}{(\mathrm{Im}\,\tau)^2} = \frac{\pi}{3} \tag{2.32}$$

we find the one-loop renormalization of the Planck mass for type-II string theory compactified on symmetric orbifolds

$$\Delta\mathcal{L}_{\mathrm{eff}} = s\frac{\pi^2}{6NM}M_s^2\sqrt{g}R + \cdots, \tag{2.33}$$

where the dots include higher derivative terms and we have recovered the string scale M_{s}.

In the case of the $\mathbb{Z}_2$ orbifold we have $(v_2, v_3, v_4) = \frac{1}{2}(1, 0, -1)$ and therefore four supersymmetries. There are then two vanishing theta functions of the type (2.25) in (2.24) and the derivative can only act on one of them. There are therefore, as expected, no sectors that contribute, and the one-loop renormalization of the Planck mass vanishes.

With the values of the GUT scale $M_{\mathrm{GUT}} \approx 2\times 10^{16}$ GeV, the Planck mass $M_{\mathrm{P}} \approx 10^{19}$ GeV and $16\pi \approx 50$ (see (2.1)) we find

$$\frac{M_{\mathrm{P}}^2}{16\pi} \approx 5000 M_{\mathrm{GUT}}^2 \tag{2.34}$$

In the case of the $\mathbb{Z}_6\times\mathbb{Z}_6$ orbifold we find the one-loop renormalization of the Einstein-Hilbert action of

$$\frac{1190\pi^2}{6^3}M_{\mathrm{s}}^2 \approx 50 M_{\mathrm{s}}^2. \tag{2.35}$$

Under the assumption that the total value of the Planck mass is entirely due to one loop (i.e. the tree value is small, e.g. of the order of the GUT scale or vanishing) this would give a string scale of

$$M_{\mathrm{s}} \approx 2\times 10^{17}\mathrm{GeV} \approx 10 M_{\mathrm{GUT}}. \tag{2.36}$$

Amusingly, this value of $M_{\mathrm{s}}/M_{\mathrm{GUT}}$ is close to the one obtained at tree level using the experimental value of α_{GUT}.

According to the argument put forward in [2] a value for $M_{\rm s}/M_{\rm GUT}$ closer to 1 could result from a theory possessing a large number N of species ("flavours"). In order to find a superstring model containing an arbitrarily large N, let us consider type-IIB string theory compactified on $M_4 \times EH_3$, where M_4 is 4-dimensional Minkowski space and EH_3 is the Eguchi-Hanson space (see [11]) that is a Calabi-Yau space with 3 complex, i.e. 6 real, dimensions and is the same as the orbifold $T_6/\mathbb{Z}_3$ where the singularities have been blown up. As in [16] we consider a background of N_b coincident D5-branes wrapping a supersymmetric 2-cycle of EH_3. The total one-loop renormalization of the Planck mass is given by

$$\Delta\mathcal{L}_{\rm eff} = \left(\frac{4\pi^2}{9} + cN_b\right) M_s^2 \sqrt{g} R + \cdots \,, \tag{2.37}$$

where the first term is (2.33) with $s = 8$ for the $\mathbb{Z}_3$ orbifold and the second term is the contribution of the open strings that end on the branes that is determined by the constant c. At low energies the massless open string modes that can propagate lead to pure $\mathcal{N} = 1$ super Yang-Mills with gauge group SU(N_b) in 4 dimensions (see [16]) with gauge coupling (see [17])

$$g_{YM}^2 = 2g_s(2\pi)^{p-2}(\alpha')^{(p-3)/2}\,, \qquad p = 5\,. \tag{2.38}$$

The field theory computation using the heat kernel regularization with the string scale $M_{\rm s}$ as cut-off yields, in leading order in the coupling constant, the contribution of N_b $\mathcal{N} = 1$ vector multiplets

$$\frac{N_b}{64\pi^2} M_s^2 \sqrt{g} R \qquad \text{or} \qquad c = \frac{1}{64\pi^2}\,. \tag{2.39}$$

If we assume that the string scale is of the order of the GUT scale and that the large value of the Planck mass is entirely due to one loop, this leads to $N_b \approx 3 \times 10^6$. It does definitely not seem natural to have a configuration of such a large number of coincident branes as the vacuum state of string theory. The problem is that we have only considered the leading contribution at weak coupling for the massless spectrum (gauge fields) of the open strings that end on the branes, and not the whole tower of states. A full string computation may eventually give much larger contributions to the one-loop renormalization of the Planck mass (using the same type of field-theory computation, the result that corresponds to (2.33) is also much smaller than the full string result), but it will still have a term proportional to $N_b M_s^2 \sqrt{g} R$. The needed number of branes may correspondingly be smaller.

3. Conclusion

By considering orbifold compactifications of type-II string theory we have shown that a rather large one-loop renormalization of the Planck mass is possible, depending on the choice of compactification within the set that preserves $\mathcal{N} = 2$ space-time supersymmetry in 4 dimensions. This is in contrast to heterotic string models, where there is no renormalization for any compactification that preserves $\mathcal{N} \geq 1$ space-time supersymmetry in 4 dimensions. As discussed in the introduction, in order to be able to lower the string scale towards the GUT scale, a large number N entering in a different way gauge and gravity loops is needed. Unfortunately, string backgrounds containing such a free parameter must

JHEP10(2002)026

involve brane configurations for which adequate loop techniques are still lacking. We hope, however, to have shown that a parametrically large renormalization of the Planck mass (in string units) is all but impossible.

Acknowledgments

I thank Gabriele Veneziano for suggesting this problem and for many useful discussions and ideas.

References

[1] K.R. Dienes, *String theory and the path to unification: a review of recent developments*, *Phys. Rept.* **287** (1997) 447 [hep-th/9602045].

[2] G. Veneziano, *Large-N bounds on and compositeness limit of, gauge and gravitational interactions*, *J. High Energy Phys.* **06** (2002) 051 [hep-th/0110129].

[3] E. Kiritsis and C. Kounnas, *Infrared regularization of superstring theory and the one loop calculation of coupling constants*, *Nucl. Phys.* **B 442** (1995) 472 [hep-th/9501020].

[4] E. Kiritsis and C. Kounnas, *Curved four-dimensional space-times as infrared regulator in superstring theories*, *Nucl. Phys.* **41** *(Proc. Suppl.)* (1995) 331 [hep-th/9410212].

[5] E. Kiritsis and C. Kounnas, *Infrared-regulated string theory and loop corrections to coupling constants*, in proceedings *String '95, future perspectives in string theory*, Los Angeles, CA, 1995, hep-th/9507051.

[6] E. Kiritsis and C. Kounnas, *Instabilities in strong magnetic fields in string theory*, in proceedings *Conf. on gauge theories, applied supersymmetry and quantum gravity*, Leuven, 1995 [hep-th/9509043].

[7] P.M. Petropoulos and J. Rizos, *Universal moduli-dependent thresholds in $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds*, *Phys. Lett.* **B 374** (1996) 49 [hep-th/9601037].

[8] E. Kiritsis, C. Kounnas, P.M. Petropoulos and J. Rizos, *On the heterotic effective action at one-loop, gauge couplings and the gravitational sector*, in proceedings 5^{th} *Hellenic School and Workshop on Elementary Particle Physics*, Corfu, Greece, 1995 [hep-th/9605011].

[9] P.M. Petropoulos, *One loop corrections to coupling constants in string effective field theory*, in proceedings 5^{th} *Hellenic School and Workshop on Elementary Particle Physics*, Corfu, Greece, 1995 [hep-th/9605012].

[10] E. Kiritsis, C. Kounnas, P.M. Petropoulos and J. Rizos, *Universality properties of $N = 2$ and $N = 1$ heterotic threshold corrections*, *Nucl. Phys.* **B 483** (1997) 141 [hep-th/9608034].

[11] J. Polchinski, *String theory volume I and II*, Cambridge University Press, Cambridge 1998.

[12] D. Bailin and A. Love, *Orbifold compactifications of string theory*, *Phys. Rept.* **315** (1999) 285.

[13] J.A. Minahan, *One loop amplitudes on orbifolds and the renormalization of coupling constants*, *Nucl. Phys.* **B 298** (1988) 36.

[14] I. Antoniadis, E. Gava and K.S. Narain, *Moduli corrections to gravitational couplings from string loops*, *Phys. Lett.* **B 283** (1992) 209 [hep-th/9203071].

[15] K. Förger, B.A. Ovrut, S.J. Theisen and D. Waldram, *Higher-derivative gravity in string theory*, *Phys. Lett.* **B 388** (1996) 512 [hep-th/9605145].

[16] P. Di Vecchia, A. Lerda and P. Merlatti, *$N = 1$ and $N = 2$ super Yang-Mills theories from wrapped branes*, hep-th/0205204.

[17] P. Di Vecchia, *An introduction to AdS/CFT correspondence*, *Fortschr. Phys.* **48** (2000) 87 [hep-th/9903007].